A PLAN FOR THE DEVELOPMENT OF INERTIAL RECONSTRUCTION OF INITIAL STATE CLOCK (IRIS)

Ernest G. Kimme, Ph.D.
Cobit, Inc.

ABSTRACT

Increasing utilization of wideband low energy/Hertz signalling techniques in the electronic communications and allied arts is creating an urgent need for significant improvements in the stability characteristics of small, rugged, low-cost time standards. One approach to obtaining such improvements is to apply currently available sophisticated signal processing techniques to the homely crystal oscillator. The uncompensated crystal oscillator exhibits performance limitations due explicitly and directly to the physical and electrical characteristics of the cyrstal resonant element. Compensation improves the performance of the oscillator by in effect isolation of the resonator from the external environment. An alternative approach would be to attempt to create a system "image" of the resonator at a fixed point in time and synthesize the oscillator output from this fixed image. This concept suggests application of plant estimation theory to the design of a crystal-based high-performance time standard. The basic crystal can be used to provide state evolution information to a processing subsystem which simply utilizes this information to provide a statistically stable estimate of the state of the resonator at some fixed point in its history, say at "turn-on time." This paper develops the theoretical basis for such a system. Kalman filtering techniques are utilized to provide initial-state estimation, and a suitable signal synthesis technique produces the time standard output signal. The crystal oscillator runs uncompensated and is merely utilized to obtain data to present to the filter on the current state of the crystal resonator. The paper includes complete generic flow charts of the proposed state estimation process and supporting signal processing. The program of development of a prototype IRIS system is described.

1. Introduction and Motivations

Provision of stable low-noise timing signals is an essential requirement in modern instrumentation, communications, and navigation systems, which utilize highly complex signal structures. High-performance distributed and spatially flexible communications systems require some sort of functional subsystem which provides the effect of universal synchronization. System approaches to date have generally involved (locally or globally) star configurations insofar as communication of timing information is concerned, relying upon remote acquisition of timing from central high-precision sources. Environmental and cost considerations have dictated this type of approach; stable sources have been both expensive and vulnerable to extremes of external system stresses.

A common system implementation of a timing source is the electronic oscillator. The design technology for this generic device is well understood, and for the purposes of this exposition can be represented by a high-gain amplifier with a resonant feedback path, as in Figure 1-1.

The quality of this device, in the limit as gain of the amplifier is increased, is determined by the qualities of the resonator: spectral purity of the output timing signal is directly related to the resonator bandwidth, for example, and the stability of the frequency of the output is a direct function of the stability of the resonator.

The crystal oscillator, in which the resonant element is a piezoelectric crystal, is the common intermediate-technology implementation of the generic device. The rationale for this choice is a matter of history and needs no expository emphasis here. It suffices to observe that achievement of high performance qualities in such designs depends upon the characteristics of available crystal resonant elements, and that inherent physical limitations on the quality of crystal resonators translate directly to inherent performance limitations of crystal oscillators.

These physical limitations are understood in piezoelectric crystal technology in terms of the dependence of the resonance characteristics of crystals upon a set of definable physical parameters. These parameters fall into two categories, which will be oversimplified by describing them as external and internal. Internal parameters will be here considered to be those associated with the physical and chemical structure of the crystal, while external parameters will be identified with properties of the current operating environment of the crystal. Thus, e.g., crystal geometry and chemical composition are deemed internal parameters, and temperature, acceleration, and aging are deemed external.

Internal parameters of crystals are traditionally considered to be determined by the fabrication process. External parameters are managed by a variety of oscillator design techniques which are intended to attenuate the effect of these parameters upon oscillator performance. In extremis, certain external parameters are subjected to processes designed to create an artificial microenvironment for the crystal resonator in which the variation of these parameters are constrained to regions in which their effects are constant on the crystal. Temperature compensation and factory aging are examples of such techniques.

These methods increase costs of finished goods while impairing reliability and longevity; in this regard, a significant attribute of these procedures is that they are basically ad hoc processes. Conventional crystal fabrication technologies have not supported systematic application of sophisticated technologies to this problem.

Current improvements in crystal fabrication technologies have broken through this particular barrier and crystals are now being manufactured with well controlled internal parameters, resulting in increased yields and greater predictability of effects of variations in external parameters. It is, therefore, now feasible to consider borrowing methods from the established technology of system theory to provide more effective ways of dealing with these external effects. In fact, contemplation of this possibility leads to the more general idea of treating the whole crystal oscillator, not just the resonator, as a part of a total estimation and control system.

2. Applicable System-Theoretic Technologies

The ultimate system performance objective for an oscillator as a time standard is the production of an output signal which is a pure sinewave of specified frequency without any variations in that frequency. Physical difficulties in measurement of frequency immediately force compromises with this ideal and actual performance criteria seek to achieve small maximum frequency fluctuations measured over specified time intervals (long-term and short-term).

In the case of the crystal oscillator, variations of output frequency are largely produced by the resonator or crystal itself and are due to aforementioned evolution of physical characteristics of the crystal and environmentally induced variations of the external parameters. Pursuant to holistic system design philosophies, the designer may contemplate methods of controlling the effects of parameter variations rather than controlling the variations directly. A synthesized oscillator design concept emerges from this kind of reasoning, in which the designer effectively takes a snapshot of the crystal at some point in time and uses this fixed image to control a signal generation process for all subsequent time. The obvious difficulty with this idea, apart from the problems associated with measuring all relevant parameters of a crystal at a fixed point in time, is that any signal generation process ultimately requires a time base; physical intuition then strongly suggests that the quality of the synthesized timing signal will not be better than that of the processing time base.

Digital signal processing technology, however, has developed synthesizing algorithms which are largely independent of their time base signals. "Time-tagged" processing algorithms do, in fact, require only sequential clocking and can, in principle, even run independently of real time. Time base dependence reappears, however, when the results of such processing are to be converted to real output signals. If, however, the output timing signal is synthesized as a band-limited signal from a sequence of impulse amplitudes, random sequential timing variations which remain bounded by the reciprocal of the repeat bandwidth will not appear, but non-random (i.e., slow) or very large timing deviations will produce distortions of the output which will be seen as frequency deviations. These phenomena are well-known and discussed at length in published sampled-data literature of considerable vintage.

Review of prior art in this field suggests that the problem addressed here is analogous to the problem of stabilizing and correcting inertial navigation system outputs. Gyroscopes at best vaguely resemble crystals, but the problems of correcting for their internal and external parameter variations are strongly similar, in kind if not quantitatively. The inertial system technologies suggest, in fact, that slow frequency variations of a synthesized oscillator could also be controlled by appropriate design of the synthesis algorithm if their behavior is susceptible to some form of prediction and if it is possible to introduce some form of correction utilizing inputs from a sensor of the oscillator's behavior. The oscillator itself, in this case, can provide a reference for relative or differential sensing, and careful review of the established technologies is therefore indicated.

The body of applicable mathematics which supports the design of stabilized inertial systems is known as recursive system theory and has seen much development in recent times. The general conceptual approach of this discipline involves three basic ideas:

i) Identification of internal versus observable variables.

ii) Definition of a recursive functional relation governing evolution of the internal variables.

iii) Definition of the functional dependence of the observable variables upon the internal variables.

The internal variables of this paradigm are referred to as the "states" of the system, and the recursion describing their evolution is the state transition functional of the system.

In application of these concepts to crystal oscillator design, it is to be noted that the output (observable) is a band-limited signal; the Nyquist sampling theorem is then invoked to create a sequential representation of the observable variables. A standard technical device of system theory translates this sequential representation to the internal state representation as well, and accordingly, the above three steps are followed by a fourth step:

iv) Selection of a sampling period T less than the reciprocal of the half-bandwidth of the observable function and restructuring the previously defined functional dependencies in sequential form, time-tagged at integral multiples of T.

Two final steps permit introduction of powerful mathematical methods into subsequent analyses: linearization and modeling of uncertainties. The steps are:

v) Representation of functional relations as linear functionals plus higher-order terms.

vi) Incorporation of additive stochastic terms in all functional relations modeling approximation and measurement errors.

The impact of these last two steps is, first, to make the vast machinery of matrix algebra available to the analyst, and, secondly, to bring the statistical estimation techniques to the tasks of finding design parameters and predicting system performance. Appendix A elaborates these concepts.

3. Mathematical Formulation of the Plan

The state structure of an oscillator system is modeled, following the steps of Section 2 above, in vector/matrix form as

$$x(n+1)=\Phi(n)\cdot x(n) + u(n), \quad n\geq o \tag{3.1}$$

where

$$x(n) = \begin{pmatrix} x(n;1) \\ \vdots \\ x(n;N_S) \end{pmatrix},$$

$$u(n) = \begin{pmatrix} u(n;1) \\ \vdots \\ u(n;N_S) \end{pmatrix},$$

and

$$\Phi(n) = \begin{pmatrix} \Phi(n;1,1) & \dots & \Phi(n;1,N_S) \\ \vdots & & \\ \Phi(n;N_S,1) & \dots & \Phi(n;N_S,N_S) \end{pmatrix}$$

The components of $x(n)$ are values of the N_S internal states of the oscillator at time nT (identified with the parameters of the resonator), the components of $u(n)$ are random variables evaluated at times nT of zero mean and known covariance $Q(n;i,j)=E\{u(n;i)u(n;j)\}$, and the components of $\Phi(n)$ are elements of the state transition matrix at time nT.

The measurement or observation process produces the output of the oscillator in sampled-data form; let,

$$y(n;k)=s((nN_o+k)T) \tag{3.2}$$

for $k = \emptyset, 1, \ldots N_o-1$, and $n \geq o$. N_o is arbitrary, within limits discussed later.

Then

$$y(n)=K(n) \cdot x(n)+e(n), n>\emptyset$$

where

$$y(n)= \begin{pmatrix} y(n;1) \\ \vdots \\ y(n;N_o) \end{pmatrix}, \tag{3.3}$$

are the N_o measured values of the system output produced by each state vector $x(n)$,

$$e(n)= \begin{pmatrix} e(n;1) \\ \vdots \\ e(n;N_o) \end{pmatrix} \tag{3.4}$$

are errors in measurement of this output, and

$$K(n)= \begin{pmatrix} K(n;1) & \ldots & K(n;1,N_s) \\ \vdots & & \\ K(n;N_o,1) & \ldots & K(n;N_o,N_s) \end{pmatrix} \tag{3.5}$$

is the measurement matrix effecting the $x(n) \rightarrow y(n)$ transformation.

This model assumes that the system states transition every N_o sample times of the output.

Measurement errors are assumed to be independent random variables with known covariance R( ):

$$E\{e(n)e(r)^T\}=\delta(n,r)R(n) \tag{3.6}$$

Denote this covariance matrix by R.

Specification of K and Φ are essential elements in construction of this system model. K is definable nearly ad hoc upon the assignment of the x(n), but Φ requires prior application of the methods of mathematical physics to prediction of the way in which the parameters of the crystal resonator behave in time. The combination of availability of powerful high-speed digital processing devices and development of predictable crystal fabrication processes makes such a definition of Φ feasible, and provides renewed motivation for development of the methods here proposed.

4. General Description of the Oscillator Stabilization Plan

The approach first proposed to produce a highly stable timing signal was to observe the system states at a fixed point in time and forever after synthesize a sinusoidal signal of frequency defined by that one state. A point brushed aside at that time was the form this state observation would take. In point of fact, the best measurement data available to make this determination is the crystal oscillator output itself; that is to say, the state of the system at any point in time is best estimated from the oscillator output defined for that state. Unfortunately, the presence of measurement and approximation errors prevents exact determination of the states corresponding to a given output sequence even if the K transformations are invertable, which they are not in general (since, at the very least, it may be impossible to select $N_S=N_O$).

The fact that to a degree the successive values of the states of the system are related (via Φ), suggests that the fixed-state description can be achieved via statistical techniques. The applicable theory in this case is that of linear recursive estimation (LRE), also known as Kalman filtering. From Appendix A, the following is a realization of LRE techniques applied to the model of Section 3.

Let $\hat{x}(n)$ be an estimate of the state at time (n-1)T, and let P(n)an be $N_S x N_S$ correction matrix, assumed symmetric.

Form the auxiliary matrix

$$S(n)=(K(n)P(n)K(n)^T+R(n))^{-1} \tag{4.1}$$

Then update $\hat{x}(n)$ by

$$\hat{x}(n+1)=\Phi(n)\cdot P(n)K(n)^T S(n)(y(n)-K(n)\hat{x}(n))+\Phi(n)\hat{x}(n) \tag{4.2}$$

Then update $P(n)$ by

$$P(n+1)=\Phi(n)(P(n)-P(n)K(n)^T S(n)K(n)P(n))\Phi(n)+Q(n) \tag{4.3}$$

$\hat{x}(n+1)$ is the minimum-variance estimate of $x(n+1)$ based upon $\hat{x}(n)$ and the observation of $y(n)$; the $y-K\hat{x}$ term is the prediction error.

The random process $u(n)$ is represented by the correction term of (4.2):

$$\hat{u}=\Phi(n)\cdot P(n)\cdot K(n)^T\cdot S(n)\cdot(y(n)-K(n)\hat{x}(n)) \tag{4.4}$$

If the system is known to be time-varying, the covariance of $u(n)$ can be estimated from these residuals; if $\hat{Q}(n)$ is the estimate of $Q(n)$ used in obtaining $\hat{x}(n+1)$, then

$$\hat{Q}(n+1)=\frac{n\hat{Q}(n)+\hat{u}(n)u(n)^T}{n+1} \tag{4.5}$$

provides an estimate of $\hat{Q}$ for the next recursion.

Similarly, if $\hat{R}(n)$ is a current estimate of $R(n)$, $e(n)$ is estimated by

$$\hat{e}(n)=y(n)-K(n).\hat{x}(n) \tag{4.6}$$

and an estimate of $R(n)$ for the next recursion is

$$\hat{R}(n+1) = \frac{n\hat{R}(n)+\hat{e}(n)\hat{e}(n)^T}{n+1} \tag{4.7}$$

These two error covariance estimation techniques are the mechanism through which the proposed system will have the capability of compensating the synthesis process for large changes of the environmental forces affecting the system. Thus, the system will be able, to an extent depending upon the artfulness of implementation, to responding to (at least) and correct for (as a performance objective of the design) such factors as short-term high-G forces, and G-tipover, to name just two of the more troublesome aspects of the crystal oscillator environment. Equations (4.5) and (4.7) assume uniform weighting of variance components in time; performance of the proposed system is expected to be modified by selection of more exotic weightings, as exponential.

The entire process begins with an initial estimate of x; there is no reason not to select t=o or n=o as the initial time of this process, so an initial minimum-variance estimate of x(o), based on observation of y(o) only, is computed according to

$$\hat{x}(o|o)=(K^T(o)K(o))^{-1}\cdot K^T(o)\cdot y(o) \qquad (4.8)$$

(See again Appendix A.)

It is a consequence of the theory of the LRE process that $\hat{x}(o|n)$, the minimum variance estimate of x(o) based upon the first n system transition and subsequent LRE of corresponding states, is given by

$$\hat{x}(o|n)=(\Phi(n-1)\cdot\Phi(n-2)\ldots\Phi(o))^{-1}\ \hat{x}(n) \qquad (4.9)$$

This is the "fixed state" which was to be used to control the synthesis process.

Finally, the design constructs a synthesis process, described here as K (i.e., independent of n), which operates on $\hat{x}(o|n)$ to produce the stabilized output at the n^{th} epoch:

$$\hat{y}(o|n)=K.\hat{x}(o|n) \qquad (4.10)$$

K is a constant-frequency version of the family of measurement transformations {K(n)}. K may be constructed to provide a signal whose frequency is offset from that of the internal oscillator.

Equation (4.10) describes the production of the vector of observables, which were modeled as samples of the oscillator output signal. The actual output signal of this signal generation process is constructible as a band-limited signal using the Nyquist sampling theorem. In idealized implementation,

$$S(t|n) = \sum y(o|n;k)\,\delta(t-kT) \qquad (4.11)$$

This signal is applied to a band limiting filter; the impulse response of which is

$$h(t) = \cos 2\pi f_o t \cdot \frac{\sin \frac{\pi t}{T}}{\frac{\pi t}{T}} \qquad (4.12)$$

where f_o is the center frequency of the filter.

In practice, this ideal construction is unachievable since the function h(t) is not realizable, and one of the vast collection of available approximate realizations of an ideal bandpass device will be used.

This point in the output signal synthesis process is, as was noted earlier, a critical point in this development plan. In (4.11), the signal is constructed from modulated impulses occurring at the times t=nT exactly.

The LRE process adjusts the system states once an epoch, and observable timing deviations from the {nT} sequence that persist over an epoch will contribute to the prediction error and, thus, be susceptible to correction. Short-term deviations that average out over an epoch will not be incorporated in the LRE process. A development of the Nyquist sampling theorem, however, states that when s(t) has bandwidth < 1/T,

$$S(t) = \sum_n S((n+\tau_n)T)\frac{\sin \frac{\Pi}{T}(t-(n+\tau_n)T)}{\frac{\Pi}{T}(t-(n+\tau_n)T)} \qquad (4.13)$$

in the sense of convergence in mean-square (L2), when $\{\tau_n : n\}$ is a sequence of independent random variables such that $|\tau_n| \leq 1/2$ with probability one. The designer's problem is to ensure that the sampling clock satisfies these conditions; equation (4.13) then guarantees that short-term timing deviations are removed by band limiting of the synthesized signal.

5. Implementation of the System

5.1 First Design Considerations

The foregoing discussion describes the mathematical basis for a time-signal generation system utilizing an uncompensated crystal oscillator as a control element for a digital sampled-data synthesis process. The technology is based upon that developed for and applied to the stabilization of inertial navigation systems; the proposed system has for this reason been called an "Inertial Reconstruction of Initial State" (IRIS) clock. Better names may be found, but this one must do for present documentation.

The general structure of the IRIS system has been described, and details of implementation need to be examined. The general problem of implementation of high-speed large-scale digital processed is being swept away by currently developed VHSIC technology; in fact, large-scale array processing microcircuits are a target of several of these development programs. There are, however, two problems yet to consider.

Realizations of large-scale sequential signal-processing techniques encounter significant difficulties due to the presence of systematic, or nearly so, effects of discretization:

1) A/D conversions on a large scale are generally required to be sequential and timing errors interact with the various truncation and rounding operations that must be performed.

2) Internal arithmetic operations of multiplication and division also require truncations and rounding and even with an ideal time base, systematic errors can be introduced. Intermodulation effects are common.

Signal processing technology has dealt with this problem by introducing various species of randomization. A technique exists that has several attractive features, including the potential for the ultimate utilization of fast single-bit (e.g., band limiting) digitization of signal amplitudes.

This technique is referred to here as the CNM-transform and amounts to representation of signals as linear combinations of basis elements of an orthonormal family of arithmetic binary sequences. These basis sequences are all translates of each other so that the representation can be implemented by convolutions. This technique is discussed in further detail in Cobit ØØØØ2; in that reference the application of the CNM-representation to the direct A/D conversion is described. Very high speed correlators are in current development (Ref. 3) and can ultimately be expected to find application here.

This transform process has the effect of modifying, by a fixed unitary transformation, the measurement matrices $\{K(n)\}$.

The internal clock (hereinafter referred to as the "strobe" clock) can, from the previous discussions, be implemented in a technology consistent with that used for the digital processing; a simple delay-line digital multivibrator is suggested. The strobe frequency must be significantly higher than the reciprocal of the update epoch N_0T, and possibly also higher than the reciprocal of the sample time T.

The previous discussion also indicates that management of inter-stage delay will also be a significant implementation problem; this is, however, dealt with adequately in current digital design technologies.

As said earlier, an important factor in devloping the algorithms implementing an IRIS system is the physical description of the oscillator, and particularly, of the oscillator's frequency-determining element. It is obvious that this step requires understanding and control of the significant performance-related parameters of this element. The introductory remarks cited current improvements in fabrication technologies for crystals, and application of these technologies and of the physical understanding behind them to the development of specifications of state-transition descriptions for the IRIS application will be a significant area of activity in the proposed plan.

The term "precision" will be used here to emphasize these attributes of the oscillator and its crystal resonator: "PXTL" will designate the "precision crystal" and "PXCO" will designate the "precision crystal oscillator".

5.2 The IRIS System

Figure 5-1 depicts the general plan of the IRIS clock. Figure 5-2 depicts a proposed first implementation of the strobe clock. In Figure 5-1, this clock drives the spectrum-spreading A/D conversion subsystem, and is counted down to derive sample and update epoch timing. Initial State Estimation and Signal Synthesis operations are implemented as N_s-dimensional array processors. The CNM-transform step converts the synthesized band spread representatives of the signal to signal amplitudes, which are (in pulse-amplitude modulated form) applied to the smoothing filter.

5.3 CNM Transforms (A/D)

Figure 5-3 exhibits a proposed implementation of the CNM-transform operation. The signal is ideally processed in analog form; discretization is accomplished prior to handing off the signal to the estimation process. [Ref. 1] provides theoretical background for this transform process. Due to its spectrum-spreading attributes, the subsequent estimation process needs no frequency tracking or similar bandwidth adjustments. The process is referred to as "direct sequence modulation" in the world of spread spectrum communications.

The analog multipliers of Figure 5-3 need not be true multipliers, as seen in Figure 5-4. They are there implemented as switched attenuators, with consequent linearity of response.

The strobe frequency is counted down by a fixed integer Nf_o, an "update" frequency; that is to say, the system model assumes that internal system dynamics effect a change in the values assigned to the internal states of the system once every N sample times of the oscillator output.

Appendix A describes the mathematical details of the development of initial state estimation algorithms based upon the system model assuming deterministic state transitions $u(n) \equiv o$, $\Phi(n) \equiv \Phi$). It is shown in Appendix A that this model amounts to a mathematical assignment of state transition uncertainties to the role of unknown components of the observational error $\{e(n):n\}$. This deterministic case represents a first approximation to the truth, and performance of the resulting IRIS-type system will be predictably suboptimal, particularly in the presence of large transient external effects rendering the state transition inaccurate for a few data frames. Proof of concept can be achieved, however, with these simplifying assumptions, and the estimation process described in Figure 5-6 will be used for the first exploratory efforts. Figure A3-2 of Appendix A is a detailed flowchart of this estimation process derived from the detailed estimation equation 3.7 of Appendix A.

5.4 Signal Synthesis

Referring to the system definition (5.1) above, signal synthesis is simply the matrix multiplication operation ($K\cdot(\ )$) modeling the measurement or observation process, that is, if $\hat{x}(o|n)$ is the estimate of the initial state obtained at the n^{th} observation of the PXO output, then

$$\hat{y}(o|n) = K\cdot\hat{x}(o|n) \tag{5.2}$$

represents the (CNM transform of the) output if the states had remained constant with this estimated value and the measurements were produced by the operation $K\cdot(\ )$.

5.5 CNM Transform (D/A)

In principle, the two CNM transforms are identical: they have identical sampled-data representations. However, the synthesized signal is in digital form, and applicable transform processes must be implemented digitally. Figure 5-7 exhibits a version of the form this process could assume. The subsystem design plan of Figure 5-8 is presently structured around a hypothetical microcomputer implementation. The actual hardware configuration of this subsystem may have to be considerably more complex due to speed limitations of currently available low-cost microcomputer chips. Certain economies suggest themselves as well, notably, the possibilities of time-sharing the z-multiply (D) functions and of implementing the accumulator recursively. The straightforward plan of Figure 5-7 will, however, be followed initially.

The z-multiply (D) function is exhibited in Figure 5-8. Note that this is a true digital multiplication.

Incorporation of this second transform operation into the synthesis algorithm is a considered option.

5.6 Smoothing

The smoothing process is required to achieve removal of the strobe clock transitions and minimize the effects of quantizing noise. At freqencies of interest, it seems likely that RC realizations will be entirely adequate; this filter should therefore have little impact upon size and cost parameters for IRIS-type timing sources.

6. Summary and Development Plan

The proposed IRIS development will eventually produce a working hardware prototype system. Steps toward achievement of this objective are as follows:

Phase I: Oscillator Subsystem Modeling:

Technical Objective: Develop modeling techniques for representation of oscillators (resonant elements) as linear dynamic systems of display (5.1) above.

Approach: Computer implementation of relations (5.1) and verification of accuracy thereof by comparisons of laboratory measurements and computer outputs using standard statistical criteria.

Phase II: IRIS Simulation

Technical Objective: Develop verified source codes implementing the predictor-corrector process of Figure 5-7.

Approach: Write computer code implementations of the overall predictor-corrector process following the flowchart of Figure 5-7, and combine this with the oscillator subsystem model from Phase I to produce an IRIS simulator; prepare a test and evaluation plan for concept verification and execute this plan using the IRIS simulation system.

Phase III: IRIS Prototype

Technical Objective: Construct an IRIS frequency source and obtain laboratory data on its operation; this is to be proof-of-concept IRIS realization using a precision crystal as the oscillator resonant element.

Approach: Design and fabricate interface and control equipments to permit replacement of the oscillator subsystem simulator of Phase I in the overall IRIS simulation with a real oscillator, and develop a (test and) demonstration plan for the resulting prototype system. Incorporate available high-speed digital correlators in the implementation (ICOR).

Phase IV: IRIS Improvement

Technical Objective: Extend IRIS capabilities to accommodate non-random transient external parameter variations and verify the projected performance enhancement.
(Implement the relation (4.1) and (4.2) in full generality.)

Approach: Modify the oscillator model of Phase I above to include state transition uncertainty, modify the predictor-corrector software accordingly (theoretical development and attendant modifications of the predictor-corrector algorithms), modify the test and evaluation plan as needed, and repeat the process of Phase III.

Phase V: Full-Scale IRIS Implementation

Technical Objective: Develop an advanced development model of a manufacturable IRIS device.

Approach: Replace general purpose digital computer host by microcomputer(s), using available microcomputer development aids to translate the predictor-corrector code into machine code for the microcomputer(s) selected. Develop test and evaluation plan and observe and evaluate IRIS operation.

This program, as outlined, is estimated to require between 18 months and three years depending upon applied levels of effort. The result, be it noted, is not merely an enhanced type of crystal oscillator, but an altogether novel device best described as a crystal-controlled synthesized timing standard. Furthermore, the basic techniques employed here utilize crystals as stable elements, but are clearly applicable to other types of devices, e.g., rubidium standards. In such applications, the strobing elements might be crystal oscillators and the internal signal processing bandwidths would be correspondingly higher, requiring higher-speed digital devices.

Appendix A
Initial State Estimation for Sampled Data Linear Systems: Deterministic Case

Section 1. General Model:

Let $\{x(t;i):1 \leq i \leq N_S\}$ denote the states of an (ideal) linear system. For fixed i, $x(t;i)$ is a real function of time t. Let T be a fixed sampling interval, and let for each i and n

$$x(n;i) = x(nT:i) \qquad (A1.1)$$

The linearity property is an attribute of the system states; for each n there are numbers $\{\Phi(n:i,j)\}$ for which

$$x(n+1;i) = \sum_{j=1}^{N_S} \Phi(n;i,j)\ x(n;j) \qquad (A1.2)$$

It will be convenient to adopt matrix notation: let

$$x(n) = \begin{pmatrix} x(n;1) \\ \vdots \\ x(n;N_S) \end{pmatrix} \qquad (A1.3)$$

and

$$\Phi(n) = \begin{pmatrix} \Phi(n;1,1 & \Phi(n;1,2) & \ldots & \Phi(n;1,N_S) \\ \Phi(n;2,1) & & & \vdots \\ \vdots & & & \\ \Phi(n;N_S,1) & & \ldots & \Phi(n;N_S,N_S) \end{pmatrix} \qquad (A1.4)$$

Then the state evolution relation (A1.2) can be stated as

$$x(n+1) = \Phi'(n) \cdot x(n) \qquad (A1.5)$$

The observable or output variables are $\{y(t:i):1 \leq i \leq N_O\}$, and they are sampled as above to produce

$$y(n;i) = y(nT;i) \qquad (A1.6)$$

The $\{y(n;i)\}$ are derived from the states $\{x(n;i)\}$ by a linear measurement process: there are numbers $\{K(n;i,j): 1 \leq j \leq N_S;\ 1 \leq i \leq N_O\}$ for which

$$y(n;i) = \sum_{j=1}^{N_S} K(n;i,j)\ x(n;j) \tag{A1.7}$$

Again, in matrix notation, let

$$y(n) \underset{D}{\equiv} \begin{pmatrix} y(n;1) \\ \vdots \\ y(n;N_O) \end{pmatrix} \tag{A1.8}$$

and let

$$K(n) \underset{D}{\equiv} \begin{pmatrix} K(n;1,1) & \cdots & K(n;1,N_S) \\ K(n;2,1) & \cdots & K(n;2,N_S) \\ \vdots & & \vdots \\ K(n;N_O,1) & \cdots & K(n;N_O,N_S) \end{pmatrix} \tag{A1.9}$$

Then the measurement relations are

$$y(n) = K(n) \cdot x(n) \tag{A1.10}$$

It will be assumed $\dim y(n) = N_O \neq N_S = \dim x(n)$.

If $x(o)$ is an initial vector of state, the linear system under consideration has the specification

$$\begin{aligned} x(o) &= \text{initial states} \\ x(n+1) &= \Phi(n) \cdot x(n) \quad \text{evolution} \\ y(n) &= K(n) . x(n) \quad \text{(measurement)} \end{aligned} \tag{A1.11}$$

Figure A1-1 depicts this situation.

Section 2. Time-Stationary Case

Of particular interest are those systems which are time-stationary, that is, those systems whose descriptions (A1.11) can be formulated so that $\Phi(n)$ and $K(n)$ are constant functions of n:

$$
\begin{aligned}
x(o) &= \text{initial states} \\
x(n+1) &= \Phi . x(n) \quad \text{(evolution)} \qquad\qquad \text{(A2.1)} \\
y(n) &= K . x(n) \quad \text{(measurement)}
\end{aligned}
$$

The essential feature of (A2.1) is, as seen in Figure A2-1, the absence of the explicit "clock" variable n in the evolution and measurement algorithms.

Time-stationary systems have a unique position in this regard: internal timing of the system can be decoupled from external clocks, since absolute or external time is not required for the evaluation of the internal parameters Φ and K.

In the time-stationary case, Φ is referred to as the state-transition matrix, and K is referred to as the measurement matrix.

Section 3. Estimation Under Uncertainty

The previous sections deal with deterministic linear system models. The linear case is regarded as an important object of study because it serves as an approximation to the general case. In the general case, the system of (A1.11) has the form

$$
\begin{aligned}
x(o) &= \text{initial state} \\
x(n+1) &= \Phi[x:n] \quad \text{(state transition)} \quad n>o \qquad \text{(A3.1)} \\
y(u) &= K(x;n) \quad \text{(measurement)}
\end{aligned}
$$

where $\Phi[x:n]$ denotes a vector-valued functional depending upon $x(o),\ldots,x(n)$, and $K(x;n)$ is a general vector function of the vector $x(n)$.

The linear approximation is made by noting that under (A3.1), x(n) is defined by recursion upon its history x(o),...,x(n-1), and hence without loss of generality $\Phi[x:n]$ can be asumed to depend on xz(n) only. Hence, it makes sense to ask for that linear functional of x(n), which will be temporarily denoted $\Phi_L[x:n]$, that makes

$$\Phi[x:n] - \Phi_L[x:n]$$

small for all x(n) of interest. Depending on the sense taken for the phrase "small for all x(n) of interest," $\Phi_L[x:n]$ is one or another of the species of differentials defined for functionals of a vector space into itself. If the Euclidean norm is used to define "small" and $\Phi[\ ]$ is given suitable continuity properties, $\Phi_L[x:n]$ is representable as a matrix operation on x(n), as in (A1.2).

By suitable renormalizations of the x(n), the state transition relation of (A3.1) can be written

$$x(n+1)=\Phi[x:n]=\Phi(n)\cdot x(n)+(\Phi[x:n]-\Phi(n)\cdot x(n)) \qquad (A3.2)$$

The residual term can be made small for x(n) of interest.

A similar process yields

$$y(n)=K(x;n)=K(n)\cdot x(n)+(K(x;n)-K(n)\cdot x(n)) \qquad (A3.3)$$

with the same residual argument.

The residuals are next treated as statistical rather than descriptive errors. Conceptual justifications for this step abound, but a simplistic view is that (A3.1) reflects at best the extent to which the dynamic behavior of the general system is quantitatively described, and that description is almost certainly incomplete. (A3.1), therefore, can be regarded as statements made under uncertainty and the Bayesian philosophy invoked to introduce stochastic terms.

In the sense of these remarks, the system of (A3.1) is describable as a linear system with additive random components, thus:

$$\begin{aligned} x(o) &= \text{initial state} \\ x(n+1) &= \Phi(n) \cdot x(n) + u(n) \qquad n \geq o \qquad (A3.4) \\ y(n) &= K(n) \cdot x(n) + e(n) \end{aligned}$$

where {u(n)} and {e(n)} are taken to be random variables of specifiable distribution. The deterministic cases of the previous sections correspond to $u(n) \equiv e(n) \equiv o$, $n \geq o$.

Figure A3-1 depicts the general recursive linear system model under uncertainty.

Under very general conditions on the random variables u(n) and e(n), the system of Figure A3-1 is equivalent, in the sense of convergence in the mean, to a time-stationary system, under uncertainty. As noted in the previous section, this implies that the limiting behavior of the general linear system can be studied in terms of an equivalent system for which $\Phi(n) \equiv \Phi$ and $K(n) \equiv K$ for all n. (In the deterministic case,

$$\Phi = \lim_{N \to \infty} \left(\prod_{k=o}^{N} \Phi(k) \right)^{1/N+1}$$

as would be expected. The corresponding relation for K is more complicated.)

The problem of estimating the state of the system is generally solved: Given an initial state x(o), the minimum-variance estimate of x(n), denoted by $\hat{x}(n)$, based upon the measurements y(o),...,y(n), is updated to a minimum-variance estimate of x(n+1) by the generalized Kalman-Bucy relations, otherwise known as the minimum-variance linear recursive estimator (LRE). If {u(n)} are independent with covariance Q(u) and {e(n)} are independent with covariance R(n), then:

Given $\hat{x}(n)$, $y(n)$, $\Phi(n)$, $K(n)$, and $P(n)$:

$$S(n) \underset{D}{\equiv} (K(n)P(n)K(n)^T+R(n))^{-1} \qquad \text{(A3.5)}$$

$$P(n+1) = \Phi(n)(P(n)-P(n)^T K(n)^T S(n)K(n)P(n))\Phi(n)^T+Q(n)$$

$$\hat{x}(u+1) = \Phi(n)(\hat{x}(n)+P(n)K(n)^T S(n)(y(n)-K(n)\hat{x}(n)))$$

The matrix P(n) is symmetric; its initial value is $P(o) = E\{x(o) \cdot x(o)^T\}$, which can be replaced by $x(o) \cdot x(o)^T$ for computational purposes. Note that (A3.5) supposes that $\hat{x}(n)$ is used to find a predicted y(n), and that the error of this prediction (the term $y-K\hat{x}$ in the equation for $\hat{x}(n+1)$) is used to correct the estimate of the new state $\hat{x}(n+1)$, which otherwise would be simply $\Phi(n)\hat{x}(n)$. It is for this reason that the LRE is sometimes referred to as a predictor-corrector filter,

The equations (A3.5) are susceptible of considerable algebraic manipulation and can be found in many different forms. The form used here is that given in reference [2].

Section 4. Time-Stationary Systems with Deterministic State Transition

Time-stationarity, as noted previously, implies $\Phi(n)\equiv\Phi$ and $K(n)\equiv K$. It is also possible to incorporate the state uncertainty $u(n)$ into the measurement error $e(n)$ in (A3.4) and set $u(n)\equiv o$. The LRE then corrects the estimated states suboptimally, so that, although the system model is conceptually equivalent, the state estimation process will not have the minimum-variance property.

The equations (A3.5) simplify, however, and this fact makes study of this special case attractive as an intermediate step to achieving the full benefit of (A3.5). Let $\Phi(n)\equiv\Phi$, $K(n)\equiv K$; if $u(n)\equiv o$, then $Q(n)\equiv o$. If the measurement process is modified to effect randomization of the measurement error, and these errors are already time-stationary, then

$$R(n) = E\{e(n)e(n)^T\} = \sigma^2 I \qquad \text{(A3.6)}$$

where I is the identity matrix of appropriate dimension, and σ^2 is independent of n. With these relations, (A3.5) becomes:

Given $\hat{x}(n)$, $y(n)$, Φ, K, and $P(n)$:

$$S(n) \underset{D}{=} (KP(n)K^T)^{-1} \qquad \text{(A3.7)}$$

$$P(u+l) = \Phi(P(n)-P(n)K^T S(KP(n))\Phi^T+\sigma^2 I$$

$$\hat{x}(n+1) = \Phi(\hat{x}(n)+P(n)K^T S(n)(t(n)-K\hat{x}(n)))$$

Let $P(n) = P(n)/\sigma^2$; then,

$$S(n) = (K\ P(n)K^T)^{-1} = (K\sigma^2 P(n)K^T)^{-1} = \frac{1}{\sigma^2}(K\ P(n)K^T)^{-1}$$

$$\bar{\bar{D}} \quad \frac{1}{\sigma^2} \quad S(n) \tag{A3.8}$$

The recursion on P becomes

$$P(u+1)=\sigma^2 P(n+1)=\Phi(P(n)\sigma^2-\sigma^2 P(n)\cdot K^T\cdot\frac{1}{\sigma^2}S(n)\cdot K\cdot\sigma^2 P(n))\cdot\Phi^T+\sigma^2 I$$

$$=\sigma^2(\Phi(P(n)-P(n)\cdot K^T S(n)\cdot K\cdot P(n))\Phi^T+I$$

or

$$P(n+1)=\Phi(P(n)-P(n)K^T S(n)\cdot K\cdot P(n))\Phi^T+I \tag{A3.9}$$

The corrected $\hat{x}$ becomes

$$\hat{x}(n+1)=\Phi(\hat{x}(n)+\sigma^2 P(n)\cdot K^T\cdot\frac{S(n)}{\sigma^2}(y(n)-K\hat{x}(n)))$$

(A3.10)

$$= \Phi(\hat{x}(n)+P(n)\cdot K^T\cdot S(n)(y(n)-K\cdot \hat{x}(n)))$$

The error variance σ^2 enters the recursive estimation process only at the initial step, where now

$$P(o) = \frac{x(o)x(o)^T}{\sigma^2} \tag{A3.11}$$

The relations (A3.7) will next be trivially restated in a form preparatory to computer program implementation:

Initialization: Assign σ^2, Φ, and K.

Enter x

Compute $P = (xx^T)/\sigma^2$ (A3.12)

Set $\hat{x} = x$

Algorithm: Given $\hat{x}$, y, and P.

$S = (KPK^T)^{-1}$

New $\quad \hat{x} = \Phi(\hat{x}+PK^{T}S(y-K\hat{x}))$

New $\quad P = \Phi(P-PK^{T}SKP)\Phi^{T}+I$

Figure A3-2 is a flowchart of the relations (A3.12).

The initial state is always estimated (minimum variance) from the transition matrix: if $\hat{x}(o|n)$ denotes the initial state estimate based on the observations $y(o),\ldots,y(n)$, then

$$\hat{x}(o|n) = (\Phi(o)\cdot\Phi(1)\cdot\ldots\Phi(n-1)^{-1}\cdot\hat{x}(n) \qquad \text{(A3.13)}$$

In the present case,

$$\hat{x}(o|n) = \left((\Phi)^{n}\right)^{-1}\hat{x}(n) \qquad \text{(A3.14)}$$

The initial state estimation process can be concluded in the algorithm:

Initialize: $\quad \Psi = I$

Given: $\quad \Psi \qquad$ (A3.15)

New $\quad \Psi = \Phi^{-1}\cdot\Psi$

which provides a current-time estimate $\hat{x}(o|n)$ of the initial system state.

References:

[1] Representation and Analysis Techniques of Wideband Signalling Systems, Cobit ØØØØ2, Rev. 2; CUES Workshop, 19-2Ø July 1983; ANSER Corp. Arlington, VA.

[2] Optimization by Vector Space Methods, David G. Luenberger; John Wiley and Sons, New York, NY 1969 (Chapter 4, pp. 78-1Ø2).

[3] U.S. Patent #4,2Ø5,3Ø2; ICOR Co., Bellevue, WA, May 27, 198Ø.

[4] Optimization Over Time: Dynamic Programming and Stochastic Control, Vol. I, Peter Whittle F.R.S., Wiley & Sone, New York

[5] Probability, Random Variables, and Stochastic Processes, Athanasios Papoulis, McGraw-Hill, 1965

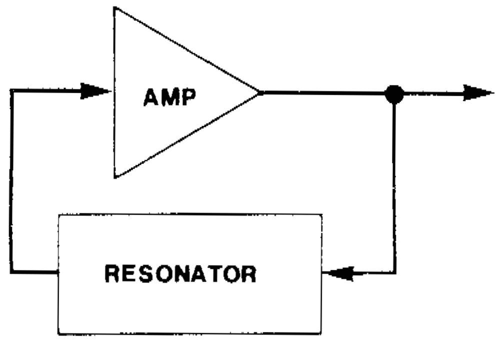

FIGURE 1-1

GENERIC OSCILLATOR

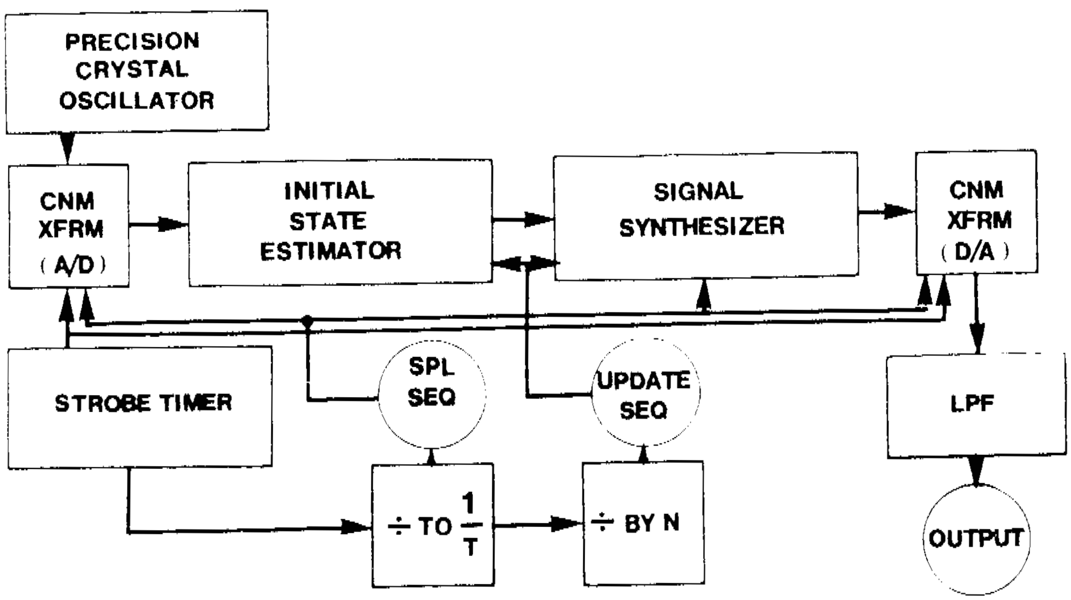

FIGURE 5-1

IRIS SYSTEM ORGANIZATION

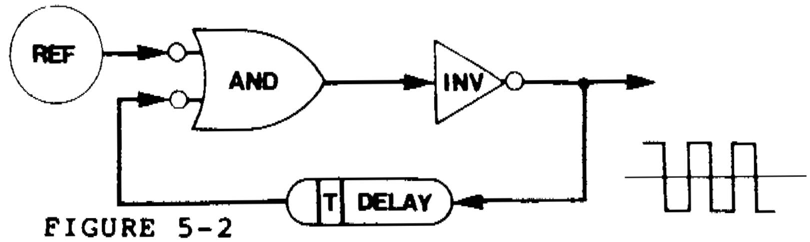

FIGURE 5-2

STROBE TIMER

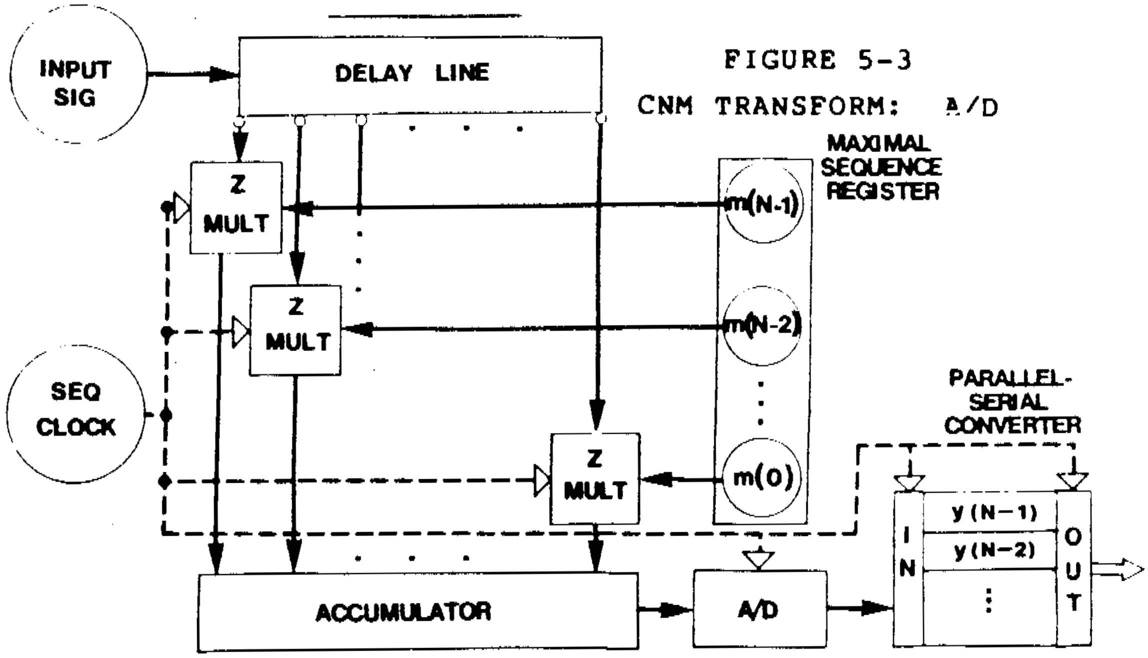

FIGURE 5-3

CNM TRANSFORM: A/D

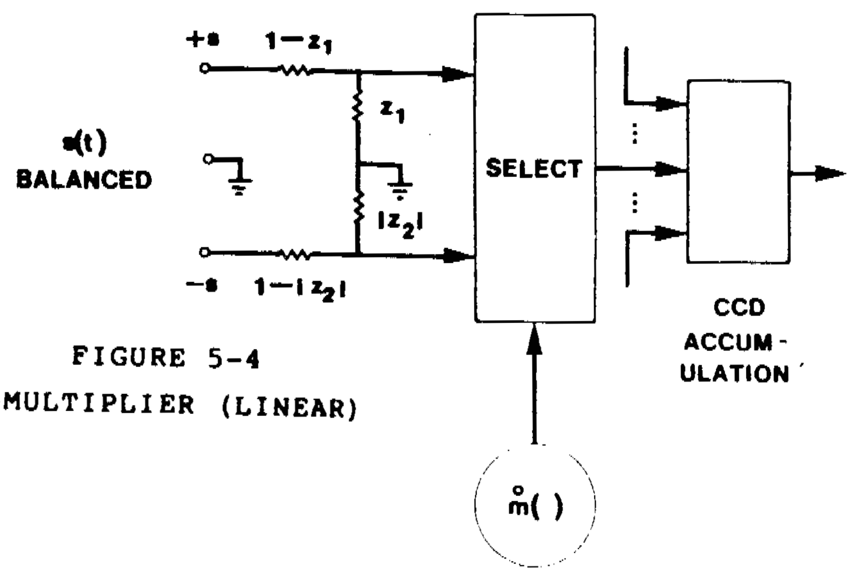

FIGURE 5-4

Z-MULTIPLIER (LINEAR)

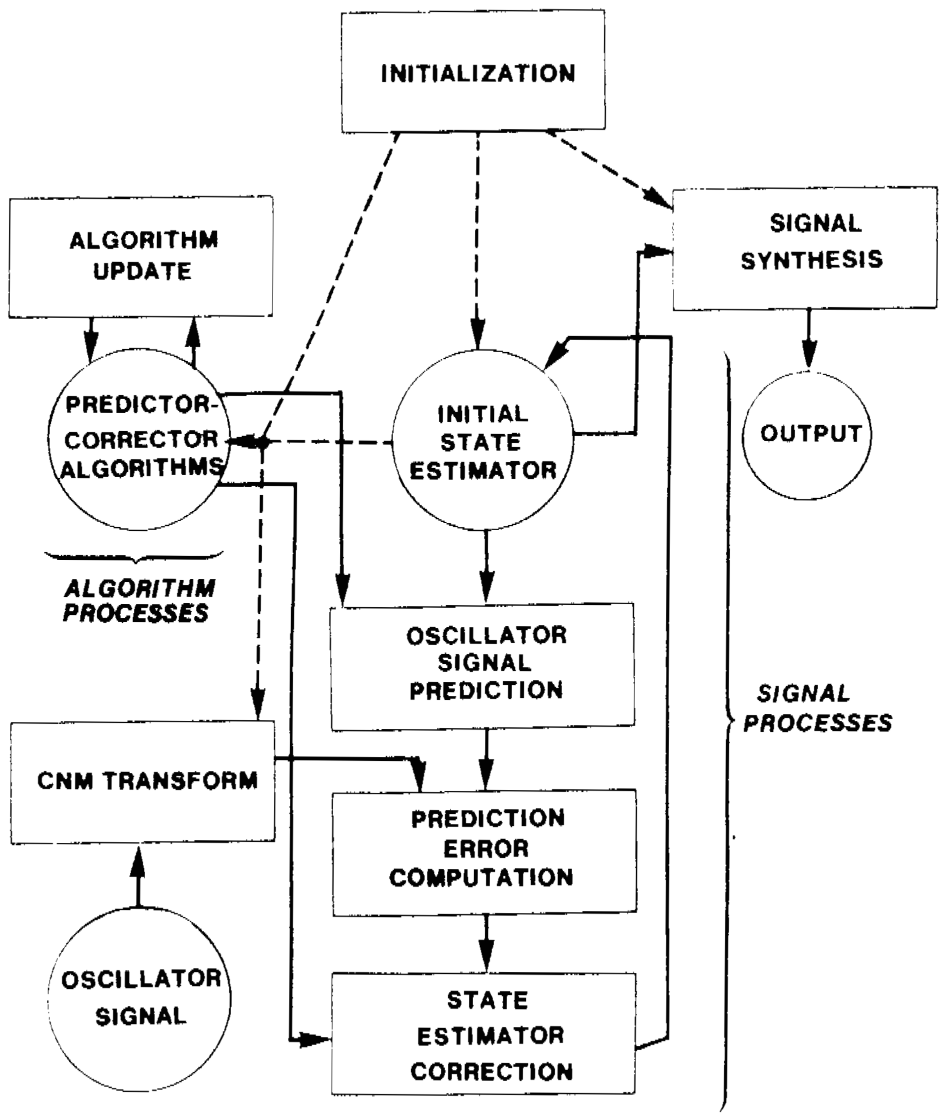

FIGURE 5-5

INITIAL STATE ESTIMATION PROCESSES

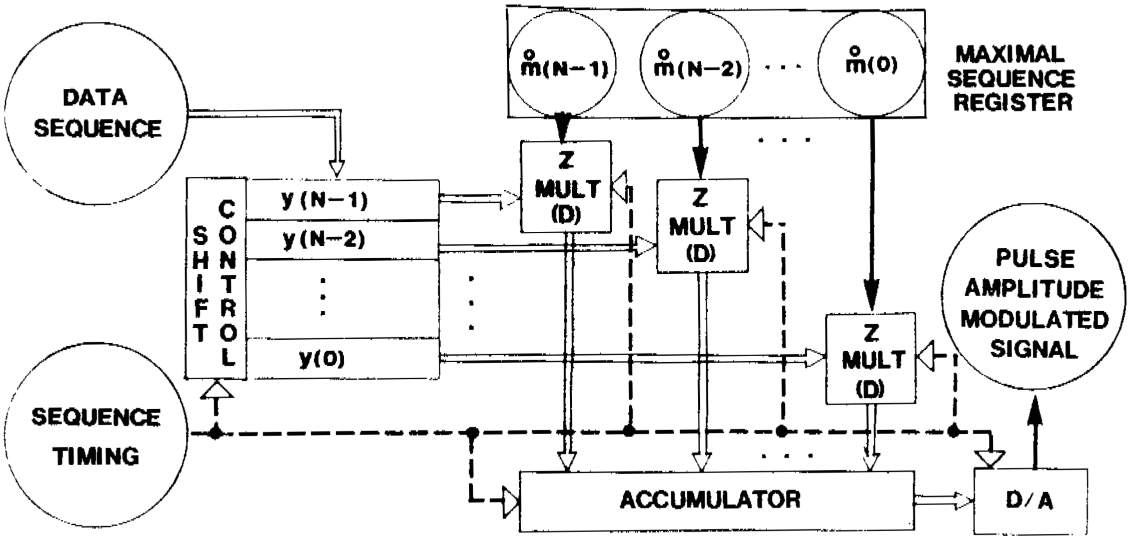

FIGURE 5-6

CNM TRANSFORM: D/A

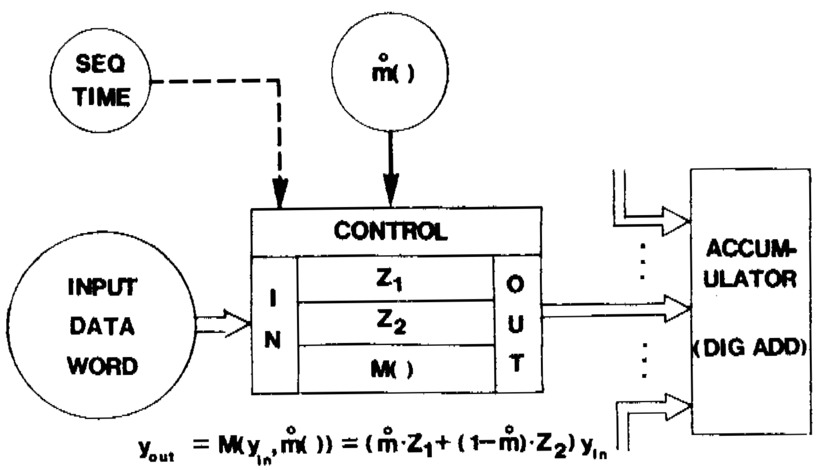

FIGURE 5-7

Z-MULTIPLIER (DIGITAL)

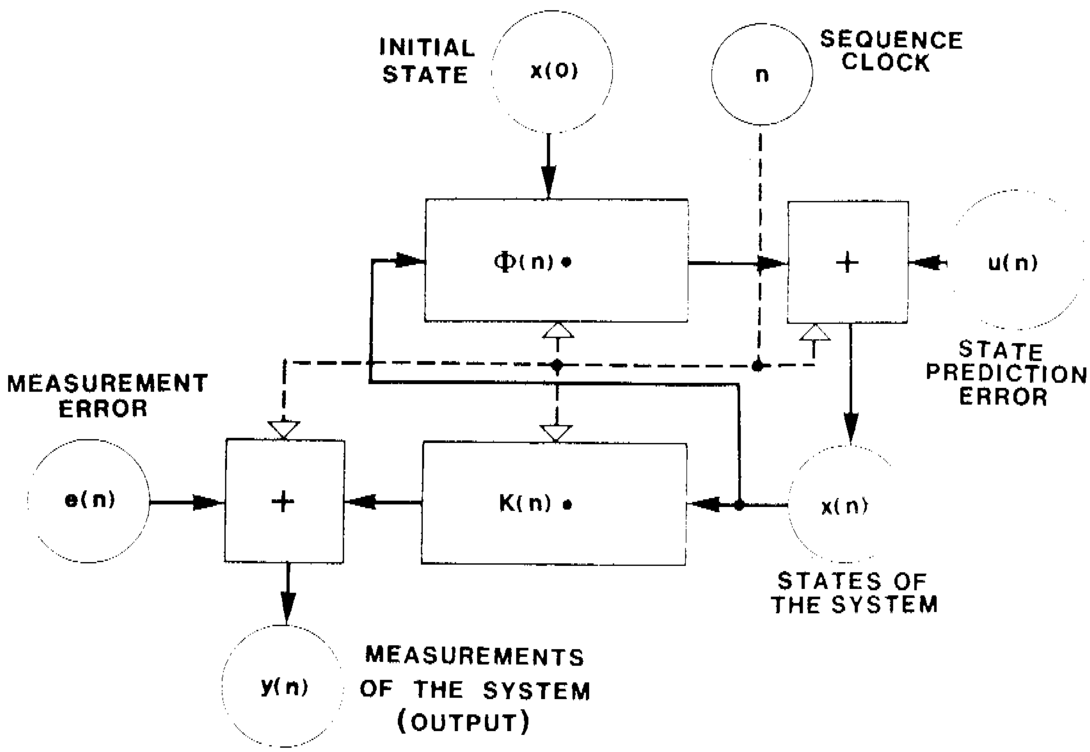

FIGURE A3-1

GENERAL RECURSIVE LINEAR SYSTEM WITH UNCERTAINTY

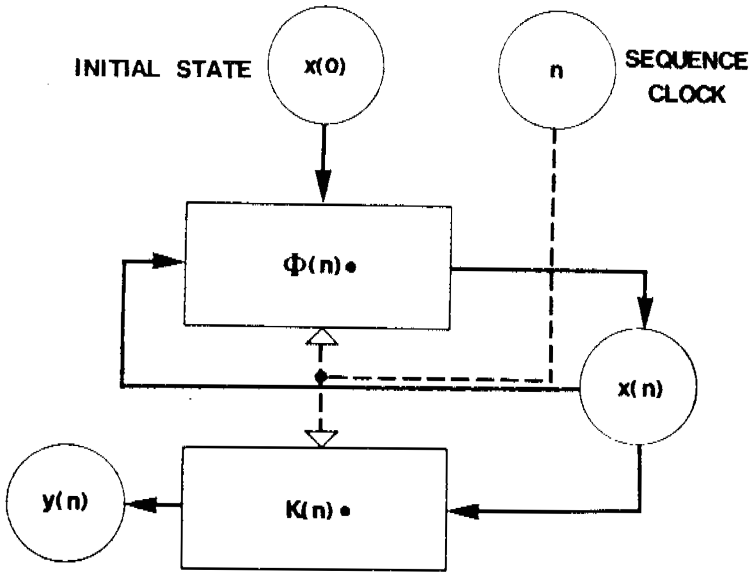

FIGURE A1-1

IDEAL LINEAR SYSTEM

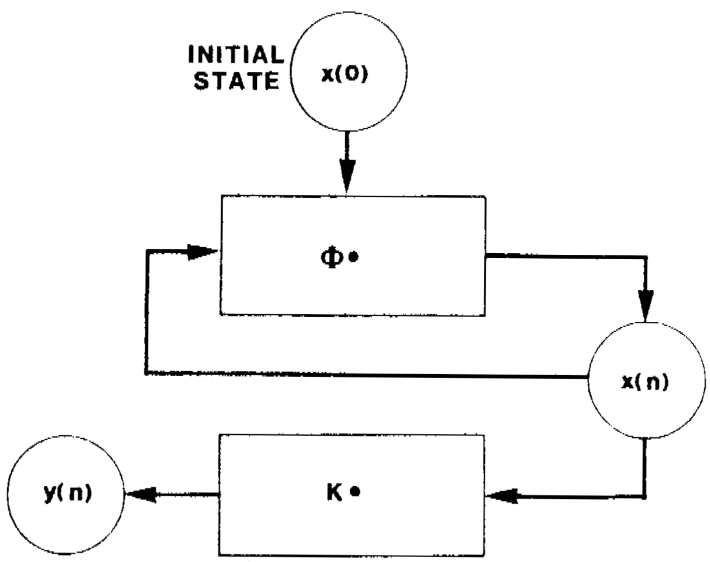

FIGURE A2-2

IDEAL TIME-STATIONARY LINEAR SYSTEM

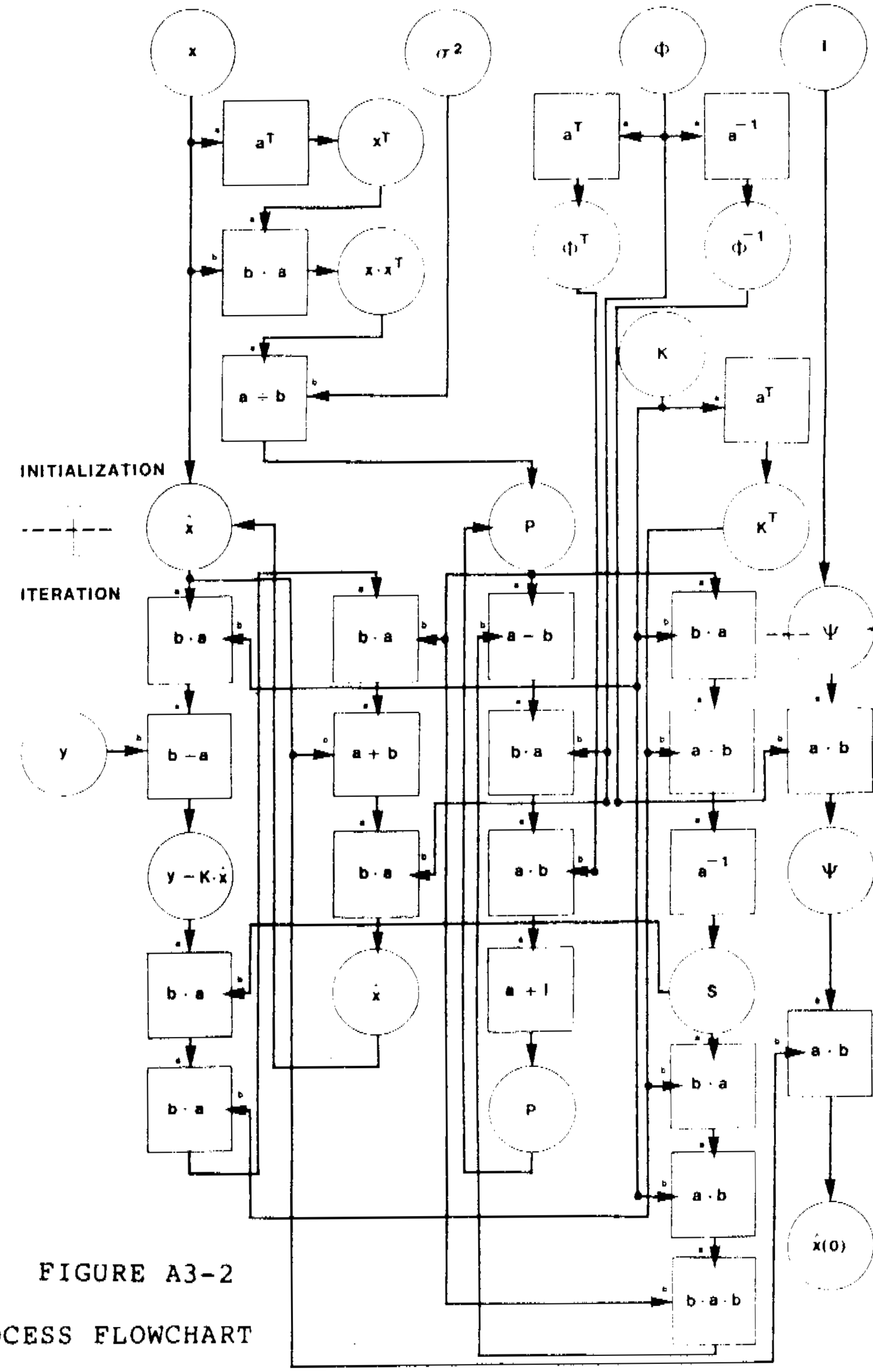

FIGURE A3-2

PROCESS FLOWCHART

QUESTIONS AND ANSWERS

DR. WINKLER:

I'm sorry, but I still do not understand why you deliberately almost, seem to avoid having your sequence timer coherent with your precision frequency source. Is there any advantage with not having it coherent? Unless for cheapness.

MR. KIMME:

I wish it to be cheap.